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$2D$ Black hole and holographic renormalization group

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Abstract: In hep-th/0311177, the Large N renormalization group (RG) flows of a modified matrix quantum mechanics on a circle, capable of capturing effects of nonsingets, were shown to have fixed points with negative specific heat. The corresponding rescaling equation of the compactified matter field with respect to the RG scale, identified with the Liouville direction, is used to extract the two dimensional Euclidean black hole metric at the new type of fixed points. Interpreting the large N RG flows as flow velocities in holographic RG in two dimensions, the flow equation of the matter field around the black hole fixed point is shown to be of the same form as the radial evolution equation of the appropriate bulk scalar coupled to $2D$ black hole.

Keywords: Black Holes in String Theory, 2D Gravity, Matrix Models, Gauge-gravity correspondence.

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Contents

1. Introduction

In a theory with gravity, the subtle issue of scale dependence is governed by the contribution of degenerate or pinched surfaces (for spherical geometry). Considering a general noncritical string background with graviton $g_{\mu\nu} = a^2(\phi)\hat{g}_{\mu\nu}(x)$ and dilaton $\Phi(\phi, x)$, the scale dependence roughly takes the form [1]

$$\beta_{\mu\nu}\frac{\delta S}{\delta g_{\mu\nu}} + \beta_\Phi \frac{\delta S}{\delta \Phi} = \text{anomaly} = \frac{1}{2} G^{IJ}\left(\phi^K(a)\right)\frac{\delta S}{\delta \phi^I}\frac{\delta S}{\delta \phi^J} + \dots\,, \tag{1.1}$$

where ϕ^Is correspond to different string states and G^{IJ} represents the metric in the space of those states, S being some appropriate low energy effective action for the boundary values of the fields at some $\phi = \phi_0$. These boundary fields $g_{\mu\nu}(x, \phi_0)$ and $\phi^I(x, \phi_0)$ are the initial data for the corresponding critical background. The effective action S is the value of the classical action on the solution with these initial data. The ellipses in the right hand side of the above expression represent local function of the fields corresponding to the string states. This anomaly arises when the world sheet completely collapses. The first term in the right hand side is due to degenerate metrics coming from the pinching of the world sheet at a given point (assuming spherical topology). This is compensated by running of the renormalized background fields to keep net scale invariance of the effective action. This is reminiscent of the *Fischler-Susskind* mechanism [2 − 7]. The equation (1.1) can be seen as Hamilton-Jacobi equation for the classical action that captures the contribution of the pinched spheres at least in one loop approximation. This also determines the sigma model beta functions for the boundary couplings $\phi^I(x, \phi_0)$

$$\beta_{ws}(\phi)^I = a\frac{\partial \phi^I}{\partial a}\,. \tag{1.2}$$

Thus the bulk evolution of the scalar fields can be interpreted as the *holographic renormalization group flows* for the boundary couplings. The running string tension $a^2(\phi)$ has an interpretation of the physical scale for the *holographic RG*.

This dynamics have been studied in the context of gravity/gauge theory correspondence in $AdS_5 \times S^5$ geometry at large 't Hooft coupling $\lambda = Ng^2_{\mathrm{YM}}$ [8−11], where the classical evolution of the scalar fields ϕ^I coupled to $5D$ bulk can be interpreted as the renormalization group flows of the $4D$ gauge theory couplings. The similar structural relationship were argued to hold in the regime of small λ too [12], where supergravity is not a good approximation for the bulk dynamics. Such a framework elevates the gravity/gauge theory correspondence to open/closed string duality. However, it is not obvious whether such a *holographic RG* exists in $2D$ noncritical background, which is an interesting question in the context of open/closed string duality.

Earlier for the $2D$ case, we calculated the boundary flows as large N renormalization group flows of matrix quantum mechanics [13]. Then extending the work in [13] we studied the nonsinglet sector of $c = 1$ matrix model by considering a gauged matrix quantum mechanics on circle with an appropriate gauge breaking term to incorporate the effect of world-sheet vortices [14]. A new coupling was introduced that would act like vortex fugacity. The flow equations indicate Berezinski-Kosterlitz-Thouless (BKT) phase transition around the self-dual radius and the nontrivial fixed points of the flow exhibit black hole like phases for a range of temperatures beyond the self-dual point. One class of fixed points interpolate between $c = 1$ for $R > 1$ and $c = 0$ as $R \to 0$ via black hole phase that emerges after the phase transition. The other two classes of nontrivial fixed points also develop black hole like behavior beyond $R = 1$. A thermodynamic study of the free energy obtained from the Callan-Symanzik equations shows that all these unstable phases do have *negative specific heat*. The thermodynamic quantities indicate that the system does undergo a first order phase transition near the Hagedorn temperature, around which the new phase is formed, and exhibits one loop finite energy correction to the Hagedorn density of states. The fixed points with negative specific heat suggest that this phase transition is associated with existence of black hole in $2D$ gravity.

Motivated by the higher dimensional picture of holographic RG flow, in this note we will give a natural interpretation of the holographic RG in $2D$ by identifying the large N renormalization group flows in the matrix quantum mechanics, derived in [13, 14], as the *radial evolution* of the scalar fields coupled to the $2D$ bulk. Here the matrix quantum mechanics acts as the boundary theory and the $2D$ bulk as the holographic dual to the boundary theory. On one hand this examines the validity of the large N RG flows derived from the matrix quantum mechanics and on the other hand illustrates an evidence of holographic RG in the context of duality between matrix model and $2D$ quantum gravity. Let us mention here that in large N RG the Callan-Symanzik equation for the physical obserbvables (which basically gives the $2D$ nonperturbative background) is essentially a world sheet hamiltonian constraint in the form of WdW equation. When computed with all the N^2 quantum mechanical degrees of freedom (that relates to having nonsinglets), this can accommodate a much richer structure [15] than a semiclassical minisuperspace WdW evolution of the background [16] and is α' exact in computing the world sheet cosmological

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constant. Whereas, the Callan-Symanzik equation for the holographic RG is a classical Hamilton-Jacobi evolution, which on being linearized and taken to the world sheet level, would reduce to minisuperspace WdW evolution of the background with a leading order effect in α'.

Of particular interest is the flow equation for compactification radius R of the Euclidian time coordinate in matrix quantum mechanics obtained in [14]:

$$\frac{dR}{dl} = -h(R)R\,, \tag{1.3}$$

where h is some function of R and becomes large and positive as we approach the fixed points with negative specific heat. In particular around black hole fixed points it is observed to diverge as $h(R) = \coth(R - R_H) + C \to \infty$, as $R \to R_H$ (radius corresponding to Hagedorn transition). The flow of R indicates a deformation of the target space geometry if one identifies the RG scale in matrix quantum mechanics to be the dilaton or the *radial direction* in the holographic picture. Starting from the simple form of the flow equation for R, near the fixed points with negative specific heat suggesting black hole like phase of the flow equations of $c = 1$ matrix model, one can derive the *cigar metric* of the $2D$ black hole [17–19]. This is particularly nice as it gives the indication that matrix quantum mechanics is capable of extracting the $2D$ black hole metric that previously had been a subject of continuous effort. It is important to note that the case with $h = 0$ for which $R = Const$ arises at the $c = 1$ fixed point [13]. The role of nonsinglet sector in matrix quantum mechanics is crucial in obtaining black hole like behavior from the bulk background.

On the other hand, from the holographic RG point of view it can be shown that the particular simple form of the R trajectory can be retrieved from the ratio of radial equations of motion of the bulk scalar fields coupled to the $2D$ background. One serious obstruction to such holographic picture for $2D$ cigar geometry is the fact that the boundary RG equations derived from matrix quantum mechanics are α' exact. The cigar background is more likely to be visible in the high curvature regime where α' is finite, while it is believed to be a Sine-Liouville background for small α' [20, 21]. Thus unlike ADS/CFT correspondence, using the dual supergravity description in the holographic RG set up should no longer be useful to make contact with the matrix quantum mechanics results and to see the cigar metric. One then needs the more general framework of open/closed duality to deal with the holographic RG for finite α' (see for example [12]). However, in this paper we observe that the R trajectory determining the cigar metric being a ratio of flow velocities of the bulk scalars, is independent of the curvature term that contains all the α' dependence. It is thus consistent to match the R trajectory determined from the Holographic RG to that derived from the α' exact computation of matrix quantum mechanics.

The paper is organized as follows. In section 2, we will briefly review the shape of the R trajectory near black hole like fixed points from worldsheet RG analysis of MQM with nonsinglet sector. In particular we will discuss the significance of the rescaling equation for the size of the compactified space in capturing the change of target space geometry around the fixed points. In section 3 we calculate the cigar metric of $2D$ black hole starting from

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the R trajectory in the large-N in matrix quantum mechanics. In section 4, via Holographic RG picture, we interpret the origin of the R trajectory in the boundary theory from the classical evolution of bulk scalars.

2. *R* Trajectory from worldsheet RG of MQM with nonsinglet sector-review

In this section we briefly review the essential ideas leading to existence of black hole fixed points in the worldsheet RG of MQM with nonsinglet sector [14] and the shape of the R trajectory near such black hole fixed points.

The large N worldsheet RG of MQM on a compactified target space [13, 14] [1] is based on the interpretation of the very existence of the *double scaling limit* as some kind of Wilsonian RG flow which is discussed in the context of $c = 0$ or pure gravity matrix models [22]. In the double scaling limit, as the matrix coupling constant $g \to g_c$, the average number of triangles in triangulations at any genus G diverges as

$$\langle n_G \rangle \sim (1-G)(\gamma_0 - 2)(1 - g/g_c)^{-1}\,, \qquad (2.1)$$

where γ_0 is the string susceptibility constant. Simultaneously with $N \to \infty$ where N is the size of the matrices or the size of the SU(N) representation to which they belong to, the regularized length of the random triangulations of the worldsheet $a \sim N^{-\frac{1}{2-\gamma_0}}$ scales to zero to keep the physical area $a^2\langle n_G\rangle \sim N^{-\frac{2}{2-\gamma_0}}(1 - g/g_c)$ or equivalently the string coupling $g_s \equiv N^2(g - g_c)^{2-\gamma_0}$ fixed. In other words a change in the regularized length scale a on the triangulated world sheet induces flow in the coupling constants of the theory in a way that one reaches the continuum limit with correct scaling laws and the critical exponents at the nontrivial IR fixed point determined by the flow equations. In the Wilsonian sense this is done by changing $N \to N + \delta N$ by integrating out some of the matrix elements, which is like integrating over the momentum shell $\Lambda - d\Lambda < |p| < \Lambda$, and compensating it by enlarging the space of the coupling constants $g_i \to g_i + \delta g_i$. For world sheet RG in MQM on a compactified target space [13] this would not only imply the evolution of the two sets of parameters of the theory (the size of the matrix N and the cosmological constant mapped into the matrix coupling g and all other matrix couplings) at the constant long distance physics with the rescaling of the regularization length, but also an additional rescaling law for the compactification radius R which we call the R trajectory.

From the results of [13] we have seen that the beta functions β_g, β_M computed by the large N RG are such that the homogenous part of the Callan-Symanzik equation indeed determines the correct scaling exponents for $c = 1$ matrix model around the nontrivial fixed point. The inhomogeneous part is related to some subtleties in the theory, like the *logarithmic scaling violation* of the $c = 1$ matrix model. From the running of the prefactor of the partition function, written in the renormalized couplings, analogous to the running due to the wave function renormalization, the free energy is observed to change sign near $R = 1$ for small value of the critical coupling [13]. This is reminiscent of the BKT

[1] A brief review of the work can be found in section 3 of [15].

transition at self-dual radius triggered by the liberation of the world-sheet vortices. The attempt in [14] was to understand the detail nature of the nontrivial fixed points of the flow that describes the physics beyond this transition. To capture the effect of vortices on the flows and the fixed points more clearly and to introduce a new coupling that would act like vortex fugacity, in [14] we analyzed the behavior of the following gauged matrix model with simple periodic boundary condition and with *an appropriate gauge breaking term*

$$
\begin{aligned}
\mathcal{Z}_N[g,\alpha,R] = & \int_{\phi_N(2\pi R)=\phi_N(0)} \mathcal{D}^{(N)^2} A_N(t)\ \mathcal{D}^{(N)^2}\phi_N(t) \\
& \exp\left[-(N)\ \mathrm{Tr}\int_0^{2\pi R} dt\ \left\{\frac{1}{2}(D\phi_N(t))^2+\frac{1}{2}\phi_N^2(t)-\frac{g}{3}\phi_N^3(t)+\frac{A_N^2}{\alpha}\right\}\right],
\end{aligned}
\tag{2.2}
$$

where the covariant derivative D is defined with respect to the pure gauge $A(t)=\Omega(t)^\dagger\dot{\Omega}(t)$ by $D\phi=\partial_t\phi+[A,\phi]$, where $\Omega(t)\in \mathrm{U}(N)$. Expanding the covariant derivative, the partition function is rewritten as

$$
\begin{aligned}
\mathcal{Z}_{N+1}[g,\alpha,R] = & \int_{\phi_{N+1}(2\pi R)=\phi_{N+1}(0)} \mathcal{D}^{(N+1)^2} A_{N+1}(t)\ \mathcal{D}^{(N+1)^2}\phi_{N+1}(t) \\
& \exp\left[-(N+1)\mathrm{Tr}\int_0^{2\pi R} dt\ \left\{\frac{1}{2}\dot{\phi}_{N+1}(t)^2+\frac{1}{2}\phi_{N+1}^2(t)-\frac{g}{3}\phi_{N+1}^3(t)\right.\right. \\
& \left.\left. +A_{N+1}(t)\ [\phi_{N+1}(t),\dot{\phi}_{N+1}(t)]+\frac{1}{2}\ [A_{N+1}(t),\phi_{N+1}(t)]^2+\frac{A_{N+1}^2}{\alpha}\right\}\right].
\end{aligned}
\tag{2.3}
$$

The $A_{N+1}(t)[\phi_{N+1}(t),\dot{\phi}_{N+1}(t)]$ term above is crucial to study the nonsinglets. Even though they are present, the gauge invariance tries to project the system to the singlet sector while the gauge breaking term prevents to do so. As the nonsinglets are confined at small temperature, such a term will have negligible effect at large radius and hence a finitely large radius representation of singlet sector can be derived without invoking such a term in the MQM action [23]. In [24], the partition function for one vortex/anti-vortex pair, i.e. in the adjoint representation was calculated by analytical continuation from the twisted partition function of the standard harmonic oscillator to that of the upside down oscillator. For $\alpha=0$, the gauge fields are forced to vanish and the partition function reduces to that of ungauged matrix quantum mechanics on circle.

Now the integration over the degrees of freedom corresponding to $(N+1)$-th row and column of the matrices $\phi(t)$ and $A(t)$ involve detail diagrammatics [14]. As in standard Wilsonian method, after evaluating the Feynman diagrams we perform following rescaling of the variables t and the conjugate momentum $1/R$ and the fields $\phi(t)$ and $A(t)$ in order to restore the original cut-off

$$
\begin{aligned}
t &\to t'(1+h\ dl)\,, \qquad R\to R'(1+h\ dl)\,, \\
\phi(t) &\to \rho\phi'(t')\,, \\
A(t) &\to (1-h\ dl)\ \eta A'(t')\,,
\end{aligned}
\tag{2.4}
$$

where,

$$dl = 1/N\,, \qquad h = h(R) + \sum_{i,j} c_{ij}\; g^i \alpha^j h_{ij}(R)\,. \tag{2.5}$$

Here ρ, η are field rescaling that can be precisely determined by considering the regularised coefficients of $\phi^2(t)/2$ and $\dot{\phi}^2(t)/2$ to be one. The parameter h appearing in coordinate rescaling is a function of the radius R and the the matrix couplings and its functional form can be explicitly determined from the behavior of the flow equations near the fixed points. In fact h turns out to be the scaling dimension of the operator coupled with the mass parameter, i.e. the coefficient of the $\phi(t)^2/2$ term, and appears in the universal term of its beta function equation. Being in the universal term of the beta function equations of the couplings g and α, the function h determines the radius at which the corresponding operators become relevant and could trigger phase transition. To summarize, h determines the scaling exponents of the fixed point by saturating itself to a constant value characteristic to that particular fixed point. Thus the rescaling of the compactification radius R in (2.4) in some sense tells us that, as the system flows to various fixed points, the target space geometry changes accordingly. This is something new in matrix model, which directly enables us to determine the target space metric around a fixed point by solving the R trajectory

$$\frac{dR}{dl} = -h(R)\; R\,, \tag{2.6}$$

in the neighborhood of the particular fixed point. For example, $h = 0$ corresponds to a $c = 1$ fixed point [13] giving a flat metric corresponding to $R = Const.$

Now (like the standard hermitian MQM on circle analyzed in [23, 13]) the MQM characterized by (2.2) with an explicit inclusion of votrices also exhibits phase transition at the self-dual radius as the free energy changes sign due to a contest between the entropy of the liberated vortices and the energy of the system [25]. The list of required beta functions (as compared to the analysis in [13]) now involves an additional flow equation β_α for the fugacity α over the usual beta functions β_g, β_M. In a range of values below the self-dual radius, a pair of fixed points given by $(g{*}^2, \alpha^*, M^* = 1)$ become purely repulsive fixed points of large coupling and exhibit *negative specific heat* and one loop correction to the Hagedorn density of states very similar to those exhibited by an unstable Euclidean black hole in flat space time. For a particular class of fixed point that flows to pure gravity ($c = 0$ matrix models) as $R \to 0$, the change of entropy exhibits a discontinuity at $R_H = 0.73$, little above the BKT temperature, indicating the Hagedorn transition to be of first order driving the system to an unstable (and possibly a black hole) phase. Around this region the involved scaling dimensions including h are large constants. As we proceed we see black hole like thermodynamics emerging from the region where h is a large positive constant [14]. The general $h(R)$ for such class of black hole fixed points is given by (See figure-7 of [14])

$$h(R) \sim \coth(R - R_H) + C\,. \tag{2.7}$$

As $R \to R_H$ The R trajectory around such black hole like fixed points thus behaves as

$$\frac{dR}{dl} \approx -\coth(R - R_H) R_H\,. \tag{2.8}$$

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In the rest of the paper, we will use this rescaling relation of the compactification radius with respect to the Liouville field, acting as the RG scale, to extract the two dimensional Euclidean black hole metric around the new type of fixed points arising above the BKT transition point discussed above.

3. Cigar metric from R trajectory near a euclidean black hole fixed point

In this section we will show how the $2D$ black hole metric can be directly obtained from the RG flow of the compactification radius R in the large N renormalization analysis of the boundary theory [14]. Recall that the rescaling of the radius R, $R \to R'(1 + h\ dl)$, to restore the original cut-off is in some sense a running of the compactification radius given by the following beta function [14]:

$$\beta_R = \frac{dR}{dl} = -h(R)\ R\,. \tag{3.1}$$

This indicates a deformation of the target space geometry. From the definition of the double scaling limit, the RG scale $l = 1/N$ is given by

$$\frac{1}{N(g-g_c)^{(\gamma_0-2)/2}} = \text{const}\,. \tag{3.2}$$

The constant in the right hand side is fixed by the closed string coupling g_s. Near the black hole fixed point, the string susceptibility exponent $\gamma_0 = 2$ [14]. It is then natural to identify the RG scale with the dilaton. This is because the regularized length $a = e^{\phi}\hat{a}$ on string worldsheet for $(2-\gamma_0) = \epsilon \to 0$ and a generic RG scale $l \sim \frac{1}{N}$ would behave as

$$a^{\epsilon} \sim e^{\epsilon\phi}\ \hat{a}^{\epsilon} \sim \epsilon l \to 0\,, \tag{3.3}$$

which implies $d\phi \sim dl$. This is similar to the case of holographic RG in AdS/CFT where the beta functions $\beta_i = d\lambda_i/d\phi$ describe the running of the boundary couplings with respect to a RG scale ϕ that is actually the scale factor of the 5D supergravity metric. Thus the beta function equation for the compactification radius becomes

$$\frac{dR}{d\phi} = -h(R)\ R\,. \tag{3.4}$$

Now, as we have seen, $h \to 0$ for the usual $c = 1$ fixed point that describes flat metric with a linear dilaton background [13]. As a result the corresponding asymptotic radius is independent of the scale. On the other hand, h saturates to a large positive value as the radius gets very small and the theory flows to the Hagedorn point $\beta \to \beta_H$ [14] where we see black hole like behavior with emergence of negative specific heat. According to the results in [14] the theory has negative specific heat indicating black hole like behavior occurring around $\beta_H = 2\pi R_H$, where R_H is just below the self dual radius (or the BKT radius).

Now as $R \to R_H$, considering the details of the divergence of h for a black hole fixed point as given by (2.7) the corresponding R trajectory (2.8) in dilaton scale can be written as

$$\frac{dR}{d\phi} \approx -\coth(R-R_H)\ R_H\,. \tag{3.5}$$

Thus the dilaton indeed solves for the cigar background

$$\phi = -\ln\cosh(R - R_H)/R_H + \phi_0 \,. \tag{3.6}$$

Thus as

$$R \to R_H \,, \quad \Rightarrow \quad \phi \to \phi_0 \,, \quad \Rightarrow \quad g_s \to e^{\phi_0} \,. \tag{3.7}$$

Note that while the cigar background (3.6) formally gives the desired asymptotic behavior

$$R \to \infty \,, \quad \Rightarrow \quad \phi \to -\infty \,, \quad \Rightarrow \quad g_s \to 0 \,, \tag{3.8}$$

the computations using (3.5) can only be trusted in the regime $R \to R_H$.

Now we will show that near $R \to R_H$, the trajectory (3.5) can compute correct conformal scale for the Euclidean $2D$ black hole metric in conformal gauge that via light cone coordinates in analytically continued space can be expressed as the two dimensional black hole metric in Kruskal-Szekeres coordinates [18, 19]. The Euclidian radius is given by $r^2 = (x - x_0)^2 + (y - y_0)^2$, which can be written in terms of the (analytically continued) light cone coordinates $u = ix + y$, $v = ix - y$ as

$$r^2 = -uv \,. \tag{3.9}$$

Using (3.9) in (3.6), the solution for ϕ in terms of the light cone coordinates near $R \to R_H$ can be expressed as

$$e^{-2(\phi-\phi_0)} = \left[\cosh(R - R_H)\right]^{2/R_H} \approx \left[1 + \frac{2}{R_H}\,\frac{1}{2}(R - R_H)^2\right] = (1 - uv/R_H) \,. \tag{3.10}$$

Now the bulk Euclidean metric in the conformal gauge taken to analytically continued space in terms of the light cone coordinates looks like

$$ds^2 = e^{2\phi}(dx^2 + dy^2) = -e^{2\phi}\; du dv \tag{3.11}$$

Rescaling $u \to \sqrt{R_H}\; u$ and $v \to \sqrt{R_H}\; v$ and putting the value of the conformal scale in terms of u v coordinates from (3.10), the metric is given by

$$ds^2 = -e^{2\phi_0}\; R_H\; \frac{du\; dv}{(1 - uv)} \sim \frac{du\; dv}{(1 - uv)} \,, \tag{3.12}$$

for $R_H \simeq 1$ and $\phi_0 = 0$ or the string coupling being of order one at the Hagedorn radius. This is nothing but the two dimensional black hole metric in Kruskal-Szekeres coordinates [18, 19]

$$ds^2 \sim -\frac{du\; dv}{1 - uv} \,, \tag{3.13}$$

with horizon at $uv = 0$ and a coordinate singularity at $uv = 1$. Thus for large positive h with the detail of the divergence given by (2.7), we have two dimensional cigar metric corresponding to the region of large negative specific heat.

4. Holographic RG origin of the *R* trajectory computing cigar background

In this section we will discuss a possible explanation of the origin of the R trajectory in the large N RG from a holographic RG perspective [8 – 10, 12, 11]. Here the matrix quantum mechanics acts as the boundary theory and the two dimensional gravity theory as the holographic dual to the boundary theory. The boundary equations in large N renormalization group flows in the matrix quantum mechanics, derived in [13, 14], can be thought of as some kind of radial evolution of the scalar fields coupled to the two dimensional gravity. In particular, from the holographic RG point of view it can be shown that the simple flow equation for R around the black hole fixed point, can be retrieved from the ratio of flow velocities (the radial equations of motion) of the bulk scalar fields coupled to $2D$ dilatonic black hole background. In other words, the $2D$ cigar black hole can be seen as the *a specific RG flow trajectory* towards nontrivial IR fixed points as seen by the boundary couplings propagating in the radial direction as the bulk scalar fields of the dual theory.[2]

4.1 Classical closed string field evolution — A review for $2D$ case

To evaluate the boundary RG flow for our $2D$ theory let us consider the scalar $\phi^I = t$ (the Euclidean time) coupled to gravity and the metric describing the space to be that of a cigar geometry

$$ds^2 = k(d\phi^2 + \tanh^2\phi \; dt^2)\,. \tag{4.1}$$

Let us consider a RG scale μ corresponding to the position $\phi = \phi_0$ in the radial direction, that separates high and low energy contributions (into local and nonlocal parts) in the effective action. A shift in the adjustable parameter ϕ_0 relates to the physical RG scale transformation through the shape of the boundary metric

$$g_{\mu\nu} = a^2(\phi)\hat{g}_{\mu\nu}\,, \qquad a = \mu/M_s\,, \tag{4.2}$$

where the RG scale a is given by the ratio of the RG energy scale μ to the string energy scale. The local and the nonlocal parts in the effective action evolves under a shift in ϕ_0 in such a way that a unique[3] classical trajectory (the field configuration on the boundary) solves the equation of motion for the total action. The flow velocities of the scalars with respect to the sliding scale ϕ_0 are proportional to the variation of the local (or equivalently the nonlocal) part of the action. The variation of the local and the non local part together solves the classical equation of motion. Using this in the classical Hamiltonian constraint, one gets a nonlinear Hamilton-Jacobi evolution for the local (or equivalently the nonlocal) part of the action which basically serves as the Callan-Symanzik equation for Holographic RG. Let us here briefly mention its relation to the Callan-Symanzik equation for the one

[2] In the context of holographic RG in ADS/CFT, a crude analogy may be drawn to the deviation of the $5d$ geometry of ADS_5 from its most symmetric form due to the evolution of the boundary couplings in the radial direction as the scalars of the dual supergravity. The deviated geometry may contain domain wall structures or naked singularity as the specific RG trajectories towards nontrivial IR QFTs [26 – 29].

[3] The uniqueness was argued in [9].

point function of the loop operator in the large N RG, namely the WdW equation, which is basically a quantized version of the hamiltonian constraint on the world sheet. The WdW equation from the large N RG being computed by a matrix quantum mechanics, is α' exact and gives a richer structure than the minisuperspace WdW when computed with N^2 quantum mechanical degrees of freedom [15]. On the other hand the Callan-Symanzik equation for the Holographic RG, on being linearized by a variation with respect to the local (or the nonlocal) part of the action, reduces to a minisuperspace WdW with a leading order in the α' dependent term.

In the following few paragraphs, we will briefly sketch few essential points of holographic RG for finite α' from [12], which will be used in the remaining part of the paper. Following [12], the total low energy effective action for small 't Hooft coupling ($\lambda = N g^2_{\rm YM}$) can be schematically written as sum over all n-loop planar open string diagrams in the closed string background ϕ

$$S(\phi) = \Gamma_0(\phi) + \sum_{n\geq 1} \lambda^n \Gamma_n(\phi)\,. \tag{4.3}$$

Here $\Gamma_n(\phi)$ represent the $(n-1)$ loop open string contribution given by the partition function of the world sheet sigma model in background given by ϕ on a sphere with n holes with all moduli parameterizing the sizes and relative locations of the holes being integrated over. Γ_0 gets contribution from sphere without holes and can be interpreted to have the same form as the standard classical action of a closed string field theory [30]. This is a finite α' generalization of the low energy effective action used in the holographic set up in the context of AdS_5/CFT_4 duality [8, 10, 11] or in warped compactifications [9] involving dynamical gravity.

Let us now talk about the issue of scale transformation in gravity with respect to the action (4.3). As a standard procedure, one introduces a cut-off a to regulate both the divergences from the sigma model expectation value and that from the integral over the moduli (when there are holes approaching each other) leading to sigma model Weyl anomaly. The renormalized background $\phi(a)$ with sigma model beta functions

$$a\frac{\partial \phi^I}{\partial a} = \beta_{ws}(\phi)^I \tag{4.4}$$

then compensates the Weyl anomaly by Fischler-Susskind mechanism [2, 3] canceling the net cut-off dependence of the total action $S(\phi(a), a)$

$$a\frac{d}{da}S(\phi(a),a) = \beta^I_{ws}(\phi)\frac{\partial S}{\partial \phi^I} + a\frac{\partial S}{\partial a} = 0\,. \tag{4.5}$$

However, for $\lambda \to 0$, this cancelation requirement is essentially the usual condition for conformal invariance as the holes due to open string loops are absent.

Now the explicit scale dependence of the total action $S(\phi(a), a)$ comes from the boundary of the moduli space described by the degenerate geometries. As pointed out in [1], in a theory with gravity the question of scale dependence or the beta function is solely determined by degenerate or pinched surfaces (assuming the overall geometry to be spherical).

This is because, all the components of the energy momentum tensor, the generator of the scale transformations, vanish or become $BRST$ commutators on gauge fixing. Thus only the degenerate geometries, that form the boundary of the moduli space, have a nonzero contribution to the scale dependence.

To evaluate the Weyl anomaly due to the explicit scale dependence of the total action, let us consider the UV regulator a on the world sheet giving a lower bound to the minimal geodesic length l_C of all non-contractible contours C surrounding a nonzero number of holes, making the boundary of the regulated moduli space to be degenerate surfaces satisfying the bound for one or more contours C . Such a pinched surface is conformally equivalent to spheres with vanishing holes separated by long cylinder of length $1/a$ for which the closed string propagator in the dual channel has acquired a large length. Cutting the cylinder and inserting a complete set of states, the partition function factorizes into a sum of products of two one-point functions defined on each half of the surface on each side of the long propagator. More specifically the evolution operator along the long tube takes the form

$$a^{L_0+\bar{L}_0} = \frac{1}{2}\sum_{I,J} |O_I\rangle G^{IJ} \langle O_J| \, . \tag{4.6}$$

This determines the anomaly to be of the form

$$a\frac{\partial S}{\partial a} = -\frac{1}{2} G^{IJ} \frac{\partial S}{\partial \phi^I} \frac{\partial S}{\partial \phi^J} \, , \tag{4.7}$$

where G_{IJ} is the $2D$ metric on the space of couplings ϕ^I. Using the form of the total action (4.3), one can see the anomaly (4.7) is only compensated by the following running of the renormalized background $\phi^I(a)$,

$$\beta^I_{ws}(\phi) = G^{IJ} \frac{\partial \Gamma_0}{\partial \phi^J} \, , \tag{4.8}$$

where $\partial\Gamma_0/\partial\phi^I$ describe the divergences due to a sphere without holes. Here one can identify Γ_0 to be the local part S_{Loc} of the low energy effective action, i.e. the Einstein part S_E, which is also related to the classical flow velocities as a function of the holographic extra dimension in the exactly same manner [8 − 11]

$$\beta^I_{ws}(\phi) = G^{IJ} \frac{\partial S_E}{\partial \phi^J} \, . \tag{4.9}$$

Plugging (4.7) in (4.5), one structurally reproduces the flow equation derived by the classical gravity[4]

$$a\frac{\partial}{\partial a} S(\phi(a), a) = \beta^I_{ws} \frac{\partial S}{\partial \phi^I} - \frac{1}{2}\left(\frac{\partial S}{\partial \phi^I}\right)^2 = 0 \, , \tag{4.10}$$

with the identification of Γ_0 to be the Einstein part of the gravitational action. Now using the flow velocities

$$\begin{aligned} \dot{\phi}^I &= \frac{1}{\sqrt{-g}} G^{IJ} \frac{\partial \Gamma_0}{\partial \phi^J} \, , \\ \frac{1}{2}(\dot{g}_{\mu\nu} - \dot{g}^\lambda_\lambda g_{\mu\nu}) &= \frac{1}{\sqrt{-g}} \frac{\partial \Gamma_0}{\partial g^{\mu\nu}} \, , \end{aligned} \tag{4.11}$$

[4]This also hold true for scalar evolution derived from classical supergravity.

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the radial evolution of the scalars in the bulk can be written as the Hamilton-Jacobi equation

$$\frac{1}{\sqrt{-g}}\left(\frac{1}{3}\left(g^{\mu\nu}\frac{\delta\Gamma_0}{\delta g^{\mu\nu}}\right)^2-\left(\frac{\delta\Gamma_0}{\delta g^{\mu\nu}}\right)^2-\frac{1}{2}\left(\frac{\delta\Gamma_0}{\delta\phi^I}\right)^2\right)=\sqrt{-g}\;\mathcal{L}(\phi,g)\,, \tag{4.12}$$

where $\mathcal{L}$ is the local Lagrangian density in the bulk.

So far, following [12], we sketched the essentials of the classical closed string field evolution from holographic RG analysis at finite α'. Let us now try to understand what does the solvent Γ_0 of the Hamilton-Jacobi equation (4.12), which is basically the local Einstein part of the low energy effective action, imply in context of our $2D$ scenario. The holographic RG essentially enables general classical trajectories of the low energy effective action of $2D$ string theory to be describable as parameter families of $1D$ fields that can change their local shape with the variation of the holographic extra dimension a (which in our example is related to the dilaton direction). Now if we consider a compactification of the other dimension such that the metric is warped (for example $2D$ cigar type of metric), the fields ϕ^I will be normalizable dynamical fluctuations of closed string modes. The local part of the low energy effective action is then the part of the bulk $2D$ action that includes the contribution of KK modes and can be described as RG trajectory on $1D$ field configuration. The classical configuration of one dimensional fields inside the manifold Σ_{Loc} is uniquely determined by the boundary values of the fields $\phi^I(a_0)$, even though the local action does not know the whereabouts of the boundary (which is consistent with the fact that in the loop expansion with finite α' (4.3) the local action Γ_0 comes from the contribution of sphere without holes).

Now the local Einstein part of the effective action would generally look like

$$\Gamma_0=S_E\sim\int\sqrt{-g}\,\left(T(\phi)+\Phi(\phi)\;R+\frac{1}{2}\,\partial^\mu\;\Phi^I M_{IJ}(\phi)\;\partial_\mu\Phi^J\right), \tag{4.13}$$

where T, Φ and M are local functions of couplings. The local action has a very similar structure as the lagrangian density $\mathcal{L}$ in the Hamilton-Jacobi evolution (4.12)

$$\mathcal{L}(\phi,g)=V(\phi)+R+\frac{1}{2}\partial^\mu\phi^I\;G_{IJ}\;\partial_\mu\phi_J\,. \tag{4.14}$$

Let us now plug (4.13) and (4.14) in (4.12) and consider the energy scale μ below the scale of cut-off $\mu_c\sim e^{\lambda r}$, where different local and nonlocal terms in the action can be separated by scaling behaviors (up to a redefinition of the nonlocal action by finite local terms). In the limit $\epsilon=\frac{\mu}{\mu_c}\to 0$, collecting the quartically divergent potential term from both sides we have

$$\frac{1}{\sqrt{-g}}\left[\frac{1}{3}\left(g^{\mu\nu}\frac{\delta\sqrt{-g}}{\delta g^{\mu\nu}}\right)^2T(\phi)^2-\left(\frac{\delta\sqrt{-g}}{\delta g^{\mu\nu}}\right)^2T(\phi)^2-\frac{1}{2}\Big(\partial_I T(\phi)\;\partial_I T(\phi)\Big)\right]=\sqrt{-g}\;V(\phi)\,, \tag{4.15}$$

which in our $2D$ case solves for

$$V(T)=-\left(\frac{1}{12}T^2+\frac{1}{2}(\nabla T)^2\right). \tag{4.16}$$

By a rescaling $\phi \to \sqrt{6}\ \phi \Rightarrow T(\phi) \to \sqrt{6}\ T(\phi)$, this boils down to

$$V(T) \to -\frac{1}{2}\Big(T^2 + (\nabla T)^2\Big)\,. \tag{4.17}$$

Recollecting (4.14) to be the two dimensional lagrangian density with spacetime potential given by (4.17) one can structurally identify T to be the $2D$ tachyon background by comparing with the target space action of $2D$ string.

We will now heuristically explain why this identification of $T(\phi)$ with the $2D$ closed string tachyon background is also consistent with the definitions of Γ_0 as the local Einstein action (4.13) as well as the classical closed string field action in the loop expansion of the total action with finite α' (4.3). Let us recall from [30] the classical closed string field action

$$\Gamma_0(\phi, \epsilon) = \frac{1}{2} < \Phi \mid Q \mid \Phi > + \sum_{n \geq 3} \frac{1}{n!}\ < \Phi^n >\,, \tag{4.18}$$

where Q is the BRST charge of the free ($\phi = 0$) sigma model and $\mid \Phi >= \sum_i \phi^i \mid \mathcal{O}_i >$ describe the states corresponding to the sigma model background Φ. These are the string fields that act as vectors in the Hilbert space of conformal field theory. Such a form of the closed string action is a remnant of the planar truncation of the higher genus expansion of the action of Riemannian surfaces embedded in two space time dimensions (which is manifest in the quantum closed string field action). Thus in $2D$, via appropriate Legender transformation, Γ_0 in (4.18) can be easily taken to the $2D$ sigma model action that is structurally same as the $2D$ local Einstein gravity (4.13) provided $T(\phi)$ is identified to be the $2D$ closed string Tachyon background.

Thus in two dimensions the local action Γ_0 in a slowly varying background will only have a local potential term $\int \sqrt{-g}\ T$, which via (4.12) is related to the spacetime potential $V(T)$ in the exactly same manner as that of a closed string tachyon field. This is another way to see that in $2D$, the nonlinear Hamilton-Jacobi equation (4.12) actually solves for the $2D$ Tachyon background. In the next section we will solve for this background. Using this background, we will then determine the flow velocities of the scalars, which will enable us to determine the R trajectory.

4.2 Solution for the $2D$ background

Now plugging the rescaling (4.2) in the radial evolution of the scalar fields approximated by the Hamilton-Jacobi equation (4.12), a classical closed string field evolution with Einstein part of the action replaced by Γ_0, we have

$$\left[\frac{a^2}{12}\left(\frac{\delta\Gamma_0}{\delta a}\right)^2 - \frac{1}{16a^2}\left(\frac{\delta\Gamma_0}{\delta a}\right)^2 - \frac{1}{2}\left(\frac{\delta\Gamma_0}{\delta\phi^I}\right)^2\right] = a^8\mathcal{L}\,. \tag{4.19}$$

One can then solve for Γ_0 with proper initial conditions and determine the beta function from (4.8). However, the main difficulty is to write down the full space time Lagrangian $\mathcal{L}$ for finite α'. Schematically, it will contain α' corrected full tachyon potential for the $2D$ space time, which we can neglect as we are looking at the pure dilatonic black hole. It will also have some general curvature term with corrections from all orders and besides that there will be all orders of derivative terms in fields (t, ϕ) . To get a crude estimation of Γ_0, let us assume slowly varying field neglecting all derivative terms and the leading curvature term $a^8\Lambda$, where Λ is the rescaled scalar curvature. Considering the shape of the metric $a \sim e^{\phi}$ and a trial solution of the form $\Gamma_0 \sim e^{4\phi}\, y(t,\phi)$, the equation (4.19) can be rewritten as,

$$\left(\frac{\partial y(\phi,t)}{\partial t}\right)^2 - \left(\frac{1}{6} - \frac{1}{8}e^{-4\phi}\right)\left(\frac{\partial y(\phi,t)}{\partial \phi}\right)^2 - \left(\frac{8}{3} - 2e^{-4\phi}\right)y(\phi,t)^2 + 2\Lambda = 0\,. \tag{4.20}$$

For large string coupling $\phi \to \infty$, which implies a large scale factor a for $2D$ noncritical theory, the $\Lambda = 0$ solution is of the form

$$y(\phi,t) = \exp\left[-\frac{4c_1}{\sqrt{6-c_2^2}}\right]\ \exp\left[\frac{4(c_2\phi + t)}{\sqrt{6-c_2^2}}\right], \tag{4.21}$$

where c_1 and c_2 are two arbitrary constants. Now choosing $c_1 = 0$ and $1/(c_2 + \sqrt{6-c_2^2}) = -2\nu/Q$, the solution for Γ_0 takes the form of the usual $2D$ Tachyon background (see the discussion at the end of the previous subsection)

$$T(t,\phi) \sim e^{-\nu t} e^{Q\phi/2}\,, \tag{4.22}$$

Q being the background charge. At large positive ν, we have zero tachyon background admitting the cigar geometry (4.1) [18, 19, 17].

4.3 The R trajectory as the ratio of flow velocities

To get the R-trajectory we will now use the $2D$ background determined above by the classical closed string field evolution and plug it in the RG flow (4.8). From there, the ratio of the pair of flow equations for the fields (t, ϕ) is given by,

$$\frac{\partial t}{\partial \phi} = \frac{\partial t/\partial \ln a}{\partial \phi/\partial \ln a} = \frac{1}{c_2} = -2\nu/Q\,. \tag{4.23}$$

In the boundary theory, $t \sim t + R$. To identify (4.23) with the boundary flow we now have to relate the background charge Q (the slope of the linear dilaton) to the compactification radius R of the target space coordinate t. Such a background charge modifies the world sheet central charge in ϕ to

$$c_\phi = 1 + 3Q^2\,. \tag{4.24}$$

Comparing the total central charge with that of the $\mathrm{SL}(2,\mathcal{R})_k/\,\mathrm{U}(1)$ coset CFT describing 2D cigar background, the linear dilaton slope is related to the asymptotic radius of the cigar geometry R_0 via the level k as

$$Q^2 = \frac{2}{k} = \frac{1}{R_0^2}\,. \tag{4.25}$$

Thus from the boundary theory point of view it will be natural to assume $Q \sim \frac{1}{R}$ in the sense that a generic compactification circle of radius R in the t direction will give rise to the compact direction of the cigar geometry in the $2D$ continuum.

On the other hand, t and $1/R$ being Fourier conjugate variables on the compactification circle, a rescaling $t \to t(1+h\,dl)$ in t is compensated by a rescaling in $1/R$ in opposite way (keeping up to $O(dl)$ term): $1/R \to (1-h\,dl)/R$ in order to keep the product intact. So the change in R due to change in the scale will be the same as that of t.

Hence the relation (4.23) leads to the same form of RG flow at the boundary as obtained in [14] by the large N renormalization group analysis

$$\frac{dR}{d\phi} \sim -2\nu R\,. \tag{4.26}$$

Moreover, as $\nu \to +\infty$ the tachyon field $T \to 0$, and the holographic RG analysis shows that for a purely dilatonic black hole the factor 2ν is large and positive in the flow equation (4.26), which is indeed the case in the boundary theory as predicted by the direct large N renormalization group analysis of the modified matrix quantum mechanics described by (2.2) [14].

One point is noteworthy here. The equation (4.26) for the compactification radius naturally arises in the RG flow of the boundary theory due to rescaling of the coordinates t, $1/R$. On the other hand, in the bulk, we are actually studying one dimensional trajectories of critical points with fixed rescaled curvature $\hat{R} = \Lambda$ traced out by the RG flow:

$$\begin{aligned}\beta^t &= \frac{\partial t}{\partial \ln a} = \frac{1}{\sqrt{-g}}\frac{\partial \Gamma_0}{\partial t}\,,\\ \beta^\phi &= \frac{\partial \phi}{\partial \ln a} = \frac{1}{\sqrt{-g}}\frac{\partial \Gamma_0}{\partial \phi}\,,\end{aligned} \tag{4.27}$$

or alternatively, one can look at the combined effect of $\partial t/\partial\phi$. From the boundary theory point of view $dt/dl \sim dR/dl$, where $dl \sim d\phi$. Hence in other words the holographic RG flow actually captures the trajectory $dR/d\phi$, that is observed to match with that of the flow in the boundary theory. The fixed points in the RG flow of the boundary theory are actually with respect to the coupling g and α of the gauged matrix model with an appropriate gauge breaking term considered in [14]. There the compactification radius R acts like a parameter. Thus the specific form of flow of R (4.26), which is a specific RG trajectory in the holographic RG point of view, describes $2D$ black hole near the black hole fixed point (characterized by negative specific heat and Hagedorn density of states) of the boundary theory given by particular fixed point values g^* and α^*. Note that this should not be confused with the fact that in the coset CFT description of the cigar geometry (4.1), the asymptotic radius is already fixed by IR regularity. The same radius is also fixed here in the integration of equation (4.26) with proper boundary condition. Also note that the issue of determining the R-trajectory from holographic RG set up, which is in the leading order in α' (using a slowly varying space time) does not affect the computation. This is because, the R-trajectory being a ratio of flow velocities is independent of the curvature term and thus of α'. Thus it is consistent to reproduce the R-trajectory computed by α' exact matrix model from holographic RG set up which is α' non exact.

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